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Selection of parameters for numerical predictions

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Data from a test on reconstituted kaolin performed under axially symmetric stress conditions in a true triaxial apparatus are used to generate sets of values of soil parameters for use with the (modified) Cam clay model. First, parameters are chosen by the traditional route, one at a time: slope of normal compression line and slope of unloading line in the compression plane, critical state stress ratio, and elastic property. This fails to take any direct account of the shear strains that occur and yet it is in order to predict the response of a soil to shearing that a model such as Cam clay is normally applied. An alternative procedure adopts an optimisation strategy to produce a simultaneous best fit set of all parameters in order to match any section or sections of the test that are reckoned to be of importance. The values of the parameters thus deduced are rather different, but the model reproduces the soil behaviour more accurately.

Introduction

The Cam clay models have become firmly established in the language of soil mechanics since their first introduction some thirty years ago (Roscoe and Schofield, 1963; Roscoe and Burland, 1968). Over the past two decades, in particular, they have been widely used in numerical analysis of geotechnical structures, especially those involving the loading of soft normally consolidated or lightly overconsolidated clays (Wroth, 1977). The Cam clay models have an important pedagogic role to play in illustrating the way in which rather simple but complete models of soil behaviour can be developed by a logical extension from consideration of ideas of yielding and plastic hardening of ductile metals (Schofield and Wroth, 1968; Muir Wood, 1990). The appeal of the Cam clay models lies in their compactness, in the very small number of soil parameters – five, plus permeability – that are necessary for a complete definition of the models, and in the physical basis of all these parameters. The Cam clay models formed a central element of a number of courses on Critical State Soil Mechanics that were presented in Britain and Europe between 1975 and 1985 (Wroth et al., 1975, 1979, 1981, 1982, 1985) and much was always made of the fact that the five parameters were not really new parameters, but familiar quantities seen in a new

 Predictive soil mechanics. Thomas Telford, London, 1993

(truer) light. Thus the slope M of the critical state line in the p':q effective stress plane is linked with the angle of shearing resistance ϕ'; the slopes λ, κ of normal compression and unload–reload lines in the v:ln p' compression plane are linked with compression and swelling indices C_c', C_s'; the location of the critical state line in the compression plane is defined by a reference specific volume Γ which can be linked with liquid limit w_L; and some second elastic property is required such as shear modulus G or Poisson's ratio ν. The model takes care of the rest.

The continuation of this sales tactic is therefore that no special tests are required to determine the values of the soil parameters: testing can continue as before and the five parameters can be picked off one by one.

The fundamental feature of these soil models – and the vital message of critical state soil mechanics – is the importance of volumetric strains and the parallel significance of change in volume and change in effective stress. These models belong to a more general class of volumetric hardening models. The models are driven by the volume changes occurring during normal compression; shear strains are deduced indirectly by introducing a family of plastic potentials (which in the Cam clay models happen to be identical to the yield loci, but which in other volumetric hardening models are not necessarily so (Mouratidis and Magnan, 1983)).

If soil parameters are being chosen in order that the model can be made to give a good general fit to a complete range of laboratory test data – particularly if these data are obtained from tests on reconstituted clays which undergo large volume changes as they are consolidated from slurry – then this volumetric basis for the parameter selection has a certain logic. If, however, the model is to be used for prediction of field response of natural soils then the volumetric response may be much less important than the distortional response, which is hardly considered during the process of parameter selection. Undrained deformation is a purely distortional process – neatly described in volumetric hardening models as the result of balancing equal and opposite recoverable and irrecoverable volumetric changes – and a strong emphasis on volumetric response in selection of parameters may in fact be particularly unhelpful in modelling undrained behaviour.

Equally, the models are strongly governed by the choice of critical state stress ratio M which describes an ultimate condition of infinite distortion. In practice, numerical predictions are required of deformations of geotechnical structures at working loads far removed from collapse conditions.

Numerical modelling is always an extrapolation from the known region of experimental data towards the unknown region of field response. This paper explores the heretical idea that the reputation of the Cam clay models could be improved still further if parameter selection were made a more interactive process, with the selector more consciously choosing experimental data from laboratory (or field) tests with stress levels, stress states and stress paths close to those for which numerical predictions were subsequently required.

Cam clay

For the purposes of this paper, the name Cam clay will be assumed to refer to the modified Cam clay model of Roscoe and Burland (1988) rather than the original Cam clay model of Roscoe and Schofield (1963). Whatever the historical origins of the two models it has been found easier to explain them through an assumed shape of yield locus and coincident plastic potential, rather than through an assumed plastic energy dissipation function and the assumption of normality. It is then natural to start with the (modified) Cam clay ellipse rather than the original Cam clay bullet.

The models are well known and do not require detailed description. Loading and unloading at constant stress ratio $\eta = q/p'$ are associated with linear response in the semi-logarithmic compression plane v:ln p', thus introducing parameters λ and κ (Fig. 1(*b*)). One-dimensional normal compression in an oedometer is a constant stress ratio loading process so that the validity of the assumption of linearity from which λ emerges can be directly assessed in routine testing. One-dimensional unloading is not a constant stress ratio unloading process so that the selection of κ and its link with swelling index C_s' are less soundly based.

Yield loci in the p':q effective stress plane are assumed to be elliptical, passing through the origin, centred on the p' axis, with the slope to the top of the ellipse given by M (Fig. 1(*a*)). The assumption of coincident yield loci and plastic potentials, together with the assumption that the soil is volumetric hardening — so that change in size of the yield loci implies irrecoverable plastic volume change — in turn implies that the soil ends with critical states at the stress ratio $\eta = M$. Evidently the existence of critical states can be assessed if laboratory tests are taken to sufficiently large distortions. Certainly failure stress ratios can be determined. Neither the shape of the yield loci, which has an essential but hidden role in all model predictions, nor the coincidence of plastic potentials and yield loci is ever actually investigated in routine testing.

The several assumptions lead to the plastic compliance relationships:

$$\delta\varepsilon_p^p = (\lambda - \kappa)[(M^2 - \eta^2)\delta p' + 2\eta\delta q]/[vp'(M^2 + \eta^2)] \quad (1)$$

$$\delta\varepsilon_q^p = (\lambda - \kappa)[2\eta\delta p' + \{4\eta/(M^2 - \eta^2)\}\,\delta q]/[vp'(M^2 + \eta^2)] \quad (2)$$

which apply whenever the changes in effective stress imply a change in size of the yield locus. The elastic compliance relationships:

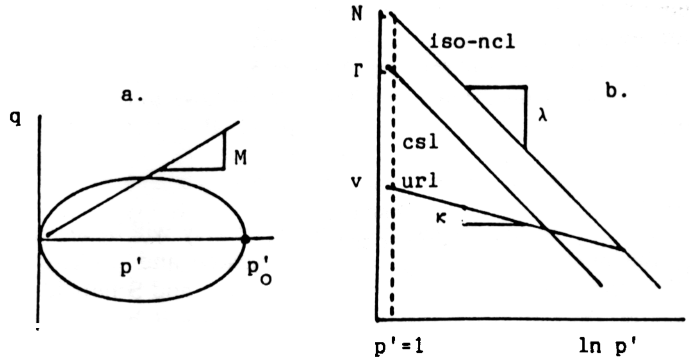

Fig. 1.(a) Elliptical Cam clay yield locus in p':q effective stress plane; (b) isotropic compression line (iso–ncl), critical state line (csl), and unloading–reloading line (url) in v:ln p' compression plane

$$\delta\varepsilon_p^e = [\kappa/(vp')]\,\delta p' \tag{3}$$

$$\delta\varepsilon_q^e = [1/(3G)]\,\delta q \tag{4a}$$

or

$$\delta\varepsilon_q^e = [2(1+\nu)\kappa]\delta q/[9(1-2\nu)vp'] \tag{4b}$$

apply for all changes in effective stress. The symbols for volumetric and distortional increments, $\delta\varepsilon_p$, $\delta\varepsilon_q$, are chosen following Calladine (1963) to indicate work conjugacy with the volumetric and distortional effective stresses p', q.

Shear modulus G, or Poisson's ratio ν, enters as a second elastic parameter, to complete the description of the isotropic elastic properties of the soil. It is recognised that with bulk modulus $K = vp'/\kappa$ proportional to mean effective stress p' it is not thermodynamically acceptable to have shear modulus also proportional to p', as is implied through the selection of a constant value of Poisson's ratio ν (Zytynski et al., 1978). This will in practice cause problems only when predictions are required of response of soils to cycles of loading and unloading.

The fifth soil parameter is required in order to be able to calculate the current specific volume v from the known stress history of the soil. If the size of the current yield locus is given by p_0' (Fig. 1(*a*)) then:

$$v = \Gamma - \lambda \ln(p_0'/2p_\Gamma') + \kappa \ln(p_0'/2p') \tag{5}$$

where Γ and p_Γ' define a reference point on the critical state line in the compression plane. Conventionally $p_\Gamma' = 1$ measured in whatever units of stress are being used. A stress of 1 kPa is extremely low for most engineering purposes. If $p_\Gamma' \simeq 4$ kPa then $\Gamma \simeq 1 + G_s w_L$, indicating the link between this reference volume Γ and liquid limit (Muir Wood, 1990). The choice of this reference stress would not be important if the v:ln p′ relationships were indeed linear over the stress range from p_Γ' to the stresses of engineering interest. Experimental evidence does not always support this (Butterfield, 1979) and since the range of mean effective stress experienced in typical geotechnical structures is not great it might be more rational to choose the value of p_Γ' to match the ambient mean effective stress and then choose Γ to fit the in situ specific volume.

Experimental data

Data to be fitted with the Cam clay model are obtained from the series of experiments performed in the Cambridge True Triaxial Apparatus (Wood and Wroth, 1972; Airey and Wood, 1988) and described by Wood (1974). These tests were performed on samples of spestone kaolin ($w_P = 0.40$, $w_L = 0.72$, 73% finer than 2 μm) consolidated in the True Triaxial Apparatus from a slurry prepared at a water content equal to twice the liquid limit. The True Triaxial Apparatus permits independent control of three principal stresses (or principal strains) without allowing any rotation of principal axes. Stress paths can be continued without interruption from consolidation to shearing (within the deformation capability of the apparatus).

The (effective) stress path of test L1 is shown in Fig. 2. It consists of anisotropic compression ($\eta = 0.3$) to $p' = 150$ kPa and unloading (OAB), followed by isotropic reloading (CDF) with two cycles of constant p′ loading and unloading (DED) with $p' = 100$ kPa; FGF with $p' = 150$ kPa. The whole test was performed with two stresses equal to each other ($\sigma_2 = \sigma_3$) and could in principle therefore have been performed in a conventional triaxial apparatus.

Proceeding incrementally, the value of λ can be deduced from the

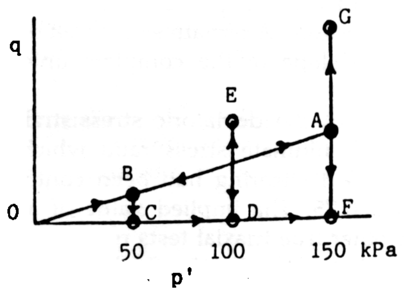

Fig. 2. Stress path of test L1: OABCDEDFGF

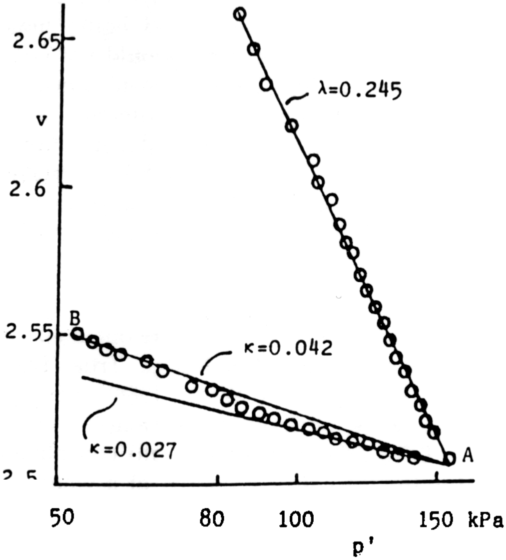

Fig. 3. Anisotropic compression and unloading in v:ln p' plane

initial anisotropic compression OA (Fig. 3) giving $\lambda = 0.245$. The value of κ can be deduced from the anisotropic unloading AB or the isotropic reloading (CD, DF). Volumetric unload–reload cycles are not the ideally elastic processes that they are assumed to be in Cam clay, and there is room for interpretation in selecting a value of κ. (Kinematic hardening models such as the 'bubble' extension of Cam clay described by Al-Tabbaa and Wood (1989) are introduced precisely to improve the match with the experimentally observed unload–reload hysteresis.) From the data shown in Fig. 3, a value of $\kappa = 0.027$ could be deduced from the initial slope of the v:ln p' relationship, immediately after the change in loading direction. Alternatively, a value $\kappa = 0.042$ could be chosen as an average slope of the complete unloading or reloading process.

Study of the shape of the deviatoric stress:strain relationship (Fig. 4(*a*)) suggests that the ultimate stress ratio which would have been reached on section FG if shearing had been continued further would have been about 0.7–0.75. The implied value of M is consistent with values observed in other true triaxial tests reported by Wood (1974).

The stress:strain relationship for the final loading stage FG is shown in Fig. 4(*a*). The initial section should, according to the Cam clay model, be purely elastic, because of the size of the yield locus which was

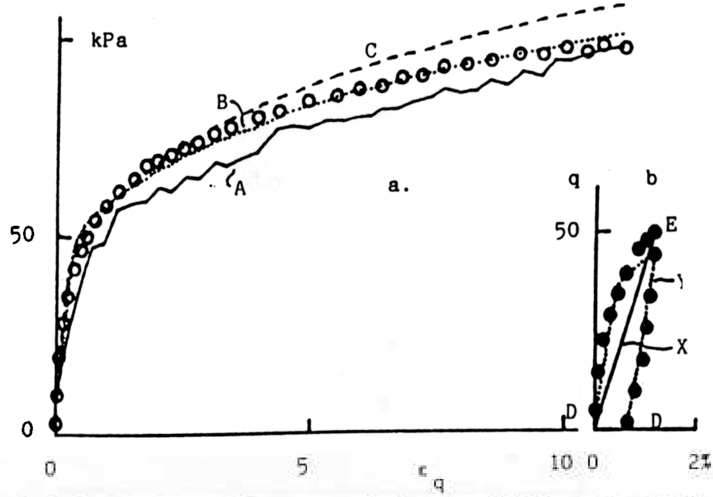

Fig. 4. Deviatoric stress:strain response for (a) stage FG (○: experiment; A: prediction based on visual selection of parameters; B: optimised fit; C: optimised fit to stage FG for $0 < \eta < 0.5$); and (b) stage DED (•: experiment; X: elastic prediction; Y: optimum fit allowing premature yield)

established during the initial anisotropic consolidation, and can be used to estimate a value of tangent shear modulus G_t = 7.5 MPa. Alternatively a secant shear modulus G_s = 3.16 MPa could be calculated for the increase of stress ratio from zero to 0.3, the entire elastic region according to the Cam clay description of this test. It is well known that for most soils the strain range over which the response is truly elastic is extremely small. However, in any situation where the plastic response of the soil is expected to become dominant, as for the soft clay being considered here, the details of the pseudo-elastic response are perhaps less important.

These values of shear modulus can be converted to equivalent values of Poisson's ratio. The measured specific volume at the start of the shearing stage FG was v = 2.479, the mean stress p' = 150 kPa. With κ = 0.027 this implies a value of bulk modulus $K = vp'/\kappa$ = 13.8 MPa. Poisson's ratio can be calculated from the relationship

$$\nu = (3K - 2G)/(6K + 2G) \qquad (6)$$

assuming that the clay is behaving isotropically. The values of Poisson's ratio are then ν_t = 0.270 using the tangent shear modulus G_t, or ν_s = 0.394 using the secant shear modulus G_s. The values of Poisson's

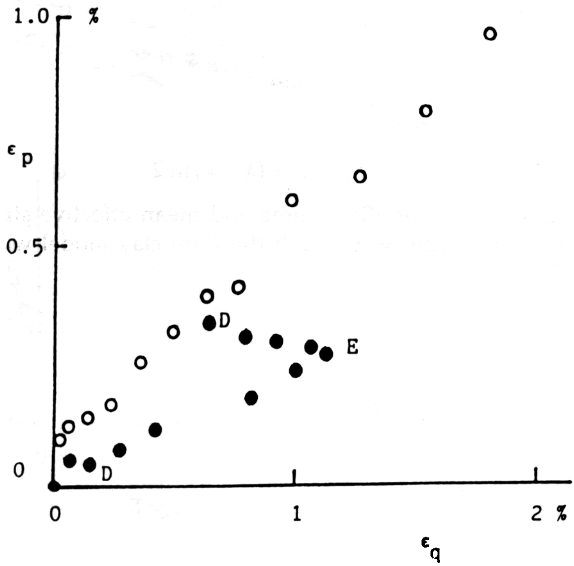

Fig. 5. Volumetric and deviatoric strain for DED (●), FG (○)

ratio would be reduced if higher values of κ were used to calculate correspondingly lower values of bulk modulus. Rather similar values of Poisson's ratio can be calculated from the intermediate loading cycle DED (Fig. 4(*b*)): the initial tangent stiffness gives $G_t = 5.0$ MPa and $\nu_t = 0.274$, the overall secant stiffness gives $G_s = 1.49$ MPa and $\nu_s = 0.424$.

Examination of the experimental data shows that the supposedly elastic cycle DED and the initial section of the loading FG are accompanied by some volumetric straining, even though the stress changes are entirely distortional ($\delta p' = 0$) ($\delta \varepsilon_p / \delta \varepsilon_q = 0.239$ for DED, $\delta \varepsilon_p / \delta \varepsilon_q = 0.505$ for FG) (Fig. 5). Such response could be described by an anisotropic elastic model such as that proposed by Graham and Houlsby (1983), but that extra refinement has not been considered here, even though the effect is clearly not insignificant.

If it is assumed that the critical state had been reached at point G, the maximum deviator stress applied during the final shearing, then the value of Γ could be calculated from the corresponding specific volume $v = 2.388$ and mean effective stress $p' = 150$ kPa. With a value of $\lambda = 0.245$, this implies $\Gamma = 3.616$. Continuing the argument of the previous section, however, it may be more useful to be able to set the value of the specific volume at the start rather than at the end of shearing. A reference specific volume can be obtained by fixing the location of the isotropic normal compression line in the compression plane:

$$v = N - \lambda \ln p' \tag{7}$$

According to the Cam clay model:

$$N = \Gamma + (\lambda - \kappa) \ln 2 \tag{8}$$

Combination of the specific volume and mean effective stress at F with the known stress history, through the Cam clay model with $\lambda = 0.245$, $\kappa = 0.027$, $M = 0.75$, leads to a value of $N = 3.739$ (which in turn implies, from (8), $\Gamma = 3.588$).

Optimisation procedure

As an alternative to direct individual estimation of values of parameters for the Cam clay model the possibility of using an automatic optimisation procedure to produce a simultaneous best fit set of parameters has been explored. Such a procedure can be adapted to ensure that the fit is obtained over the range or ranges of stress change that are expected to be relevant in a particular application — with the emphasis on the ability to match and predict response under working loads which may not approach failure.

The stress:strain response that emerges from a constitutive model is an extremely non-linear function of several model parameters. In only a very few cases will it be possible to obtain an analytical solution to the search for the optimum set of parameters, and a numerical procedure is to be preferred. The procedure adopted here is that proposed by Rosenbrock (1960), and the program used for the optimisation process has been adapted from a program written by Klisinski (1987).

The program searches for the set of n parameters that produces the minimum value of an objective function F which is a measure of the overall difference between experimentally observed and numerically predicted responses. With a given starting set of parameters, the program varies each parameter in turn in order to discover which direction in n-dimensional parameter space leads to the greatest improvement in the value of F. A new set of parameters related to the first by the direction of maximum improvement is then chosen and the procedure is repeated. The process is adaptive in that the direction of maximum improvement will in general involve variation of more than one of the n parameters: a set of n mutually orthogonal directions of progressively decreasing improvement is computed and the search for further improvement makes use of this previously determined set of directions.

The procedure moves through n-dimensional parameter space in the direction of greatest change of the function F, but since it retains

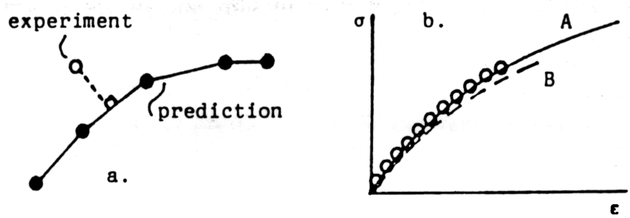

Fig. 6. (a) Objective function: shortest distance between experimental point and predicted curve; (b) alternative fits to experimental data

information from each previous step concerning the relative advantage of moving in each of the n orthogonal directions, it is more robust and more rapidly convergent than the simpler method of steepest descent. It is relatively simple and places no constraint on the nature of the objective function F. It relies on the continuity and smoothness of F but with a very irregular function it may be sensitive to starting point, and may converge on a local minimum rather than the global minimum of the function.

The choice of objective function F which is to be minimised is essentially arbitrary. Some measure of least squares fit is an obvious candidate. Klisinski (1987) uses the square root of the sum of the squares of the shortest distances from each experimental point to the piecewise linear path joining the theoretical prediction points (with appropriate scaling values to allow for the different dimensions of stress and strain) (Fig. 6(*a*)). This has a potential limitation since the closest experimental and calculated points may correspond to rather different points on the path. For example a curve (A in Fig. 6(*b*)) which stays close to the shape of the data but which is much 'longer' or 'shorter' than the experimental curve may be a 'better' fit than a curve of the correct form but slightly displaced (B in Fig. 6(*b*)), even though curve B reproduces the nature of the experimental curve rather better. To try to overcome this difficulty Klisinki adds a term to the objective function equal to the difference between the end points of the experimental and calculated paths. However, this term becomes less important as the number of data points increases, and could perhaps better be made proportional to the number of data points. This definition of objective function may also have difficulties with cyclic paths, where it is not always straightforward to identify, numerically, the relevant closest segment. Such an objective function is, however, useful when the control of the test to be predicted involves a mixture of stress and strain constraints, such as strain control of a specimen tested under conditions of plane stress, or stress control of a specimen tested under conditions of plane strain.

It has been preferred here to define the objective function directly in terms of the differences between corresponding points on the calculated and experimental paths, using experimental values of one set of quantities to control the prediction. For example, in stress driven paths the calculated path is forced to pass through all the stress points, and the objective function is simply the sum of the squares of the strain differences.

Both these objective functions have the disadvantage that sections of the path with widely spaced experimental points will receive less weighting than sections with many points. This could be overcome by weighting each increment of the objective function by some measure of the distance between adjacent points on the experimental path.

The program requires a file of the experimental data points to be fitted; a file containing the control path which provides input for the prediction; and a file which specifies the lower and upper bounds to the n parameters, initial values of these parameters, and an indication of whether each parameter is allowed to be varied as part of the optimisation procedure.

The Cam clay model is most conveniently described in terms of the strain response to changes in effective stress. The model is complete in the sense that it is able to make predictions of response in all regions of strain space — including independent variation of all three principal strains, rotation of all three principal axes. However, the structure of the model implies that not all changes in *stress* are permissible. Any attempt to cause plastic deformations with stress ratio $\eta > M$ leads to collapse of the yield surface: the soil cannot support outward stress increments, and a section of stress space (which depends on the current size of the yield surface) is thus inaccessible. It is therefore preferable to use as the control path the observed strain path even where, as for the true triaxial tests used here, the test has been conceived as a stress-controlled test, because while every strain increment implies a corresponding stress increment the converse is not always true.

Results

Although the primary objective is to improve the prediction of the model during the shear stage FG, it is of interest to observe how the optimisation procedure attempts to cope with other stages of the test. The volumetric data (specific volume and mean effective stress) from the initial anisotropic consolidation OA have been used to obtain a value for $\lambda = 0.245$. The optimisation program prefers a slightly higher value $\lambda = 0.272$, partly because the definition of objective function F implicitly gives greater weight to the data at higher stresses. The optimisation procedure for this stage can also present an opinion on the values of the other parameters because these control the link between stress ratio η

and ratio of distortional to volumetric strain $\delta\varepsilon_q/\delta\varepsilon_p$. Cam clay is not very good at getting this link correct: Muir Wood (1990) notes that Cam clay tends to predict values of earth pressure coefficient at rest K_0 which lie above Jaky's (1948) empirical expression

$$K_0 = 1 - \sin\phi' \tag{9}$$

unless simultaneous low values of both ν and $\Lambda = (\lambda - \kappa)/\lambda$ are assumed, implying dominance of the deformation by low Poisson's ratio elastic response. The optimisation program suggests $\nu = 0.37$ but $\kappa = 0.26$, implying $\Lambda = 0.04$, and $M = 0.86$.

The procedure can also be applied to the anisotropic unloading stage. The average value of $\kappa = 0.04$ for this stage is confirmed, but there is a problem with the search for the optimum value of ν (the only other parameter which has any effect during this elastic unloading). A very small positive shear strain was observed during unloading, while the deviator stress q was reducing. This pattern of response cannot be predicted with an isotropic elastic model. The best the program can do is to set $\nu \simeq 0.5$, making the shear modulus as low as possible, so that the predicted stress path shows no change in q.

The cycle of loading and unloading DED, with $p' = 100$ kPa, is expected to be purely elastic according to Cam clay, with the known stress history. The observed, typically hysteretic, shape of the stress - strain response on this cycle (Fig. 4(*b*)) clearly conflicts with this expectation, and the program, not surprisingly, is not particularly happy in trying to fit the data varying only G, or κ and ν. (Although this is a purely distortional stress path, both κ and ν are required in order to compute the value of the shear modulus.) The objective function F in this case seems to be rather flat and undulating (a Cambridge-like landscape) with a number of false minima: convergences are obtained with $\nu = 0.12$, $\kappa = 0.15$ but also with $\nu = 0$, $\kappa = 0.34$ (Fig. 4(*b*): line X).

With $M = 0.75$, the size of the yield locus created by the original consolidation OA is $p'_0 = 174$ kPa. If this known history is ignored then the observed behaviour on cycle DED can be better matched with an elastic–plastic Cam clay prediction with $p'_0 = 130$ kPa, and with $G = 3.25$ MPa, $\kappa = 0.20$, $\lambda = 0.26$, $M = 0.68$ (Fig. 4(*b*): curve Y). This is a more robust minimum to which the optimisation process is able to converge from several different starting points. Whether such a distortion of the actual history would be acceptable from an engineering point of view is a separate issue. Besides, the value of shear modulus that has been selected implies a negative value of Poisson's ratio.

The final shearing FG is best fitted with the set of parameters $G = 4.09$ MPa, $\kappa = 0.35$, $\lambda = 0.62$, $M = 0.82$ (Fig. 4(*a*): curve B). The optimisation procedure is happy to choose the size of the yield locus at the start of shearing to be $p'_0 = 178$ kPa, which is surprisingly but gratifyingly close to the value $p'_0 = 170$ kPa calculated from the known history with M = 0.82. This is again a rather robustly convergent set of parameters. It might be suggested that the value M = 0.82 gives a truer estimate of the stress ratio towards which the stress:strain response is actually heading. Again the chosen combination of shear modulus and κ implies a negative value of Poisson's ratio. If it is required to restrict the search to $\nu > 0$ then the optimum fit is obtained with $\nu \simeq 0.0$, $\kappa = 0.18$, $\lambda = 0.45$, M = 0.82. It is significant that the value of $(\lambda - \kappa)$ has remained almost the same, while the individual values have changed: it is $(\lambda - \kappa)$ that primarily controls the magnitude of the plastic distortional strain increments $\delta\varepsilon_q{}^P$ (eqn. (2)). It is particularly the value of $\Lambda = 1 - \kappa/\lambda$ that is being pulled down, indicating that improved fitting is obtained by increasing the contribution of the recoverable component of volumetric deformation. A zero or negative value of Poisson's ratio, implying a low ratio K/G

$$K/G = (2/3)(1 + \nu)/(1 - 2\nu) \tag{10}$$

is also apparently beneficial, but the present Cam clay algorithm does not permit negative Poisson's ratio to be specified.

However, if the optimisation procedure is applied only to the initial part of the shearing FG, up to stress ratio $\eta = 0.5$, then the optimum fit

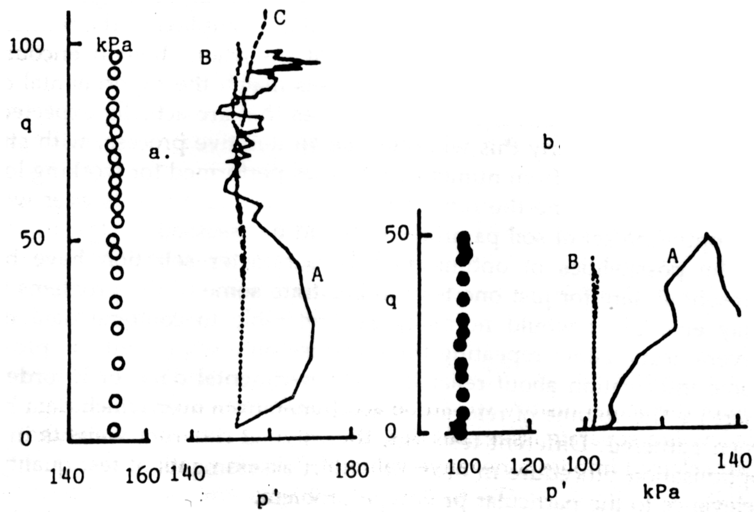

Fig. 7. p':q effective stress paths (a) FG; (b) DED (○, ●: experiment; A: prediction based on visual selection of parameters; B: optimised fits; C: optimised fit to stage FG for $0 < \eta < 0.5$)

is obtained with $G = 5.0\,\text{MPa}$, $\kappa = 0.43$, $\lambda = 0.58$, $M = 0.78$ and $p_0' = 172\,\text{kPa}$ (Fig. 4(*a*): curve C) (or $\nu \simeq 0.0$, $\kappa = 0.13$, $\lambda = 0.28$, $M = 0.80$, again with the same difference $(\lambda - \kappa)$ being chosen) – but these sets of parameters give a worse overall fit to the data of the whole shearing stage. Clearly the choice of parameters can be tuned to match the data over a selected range of interest. Both curves B and C in Fig. 4(*a*) provide a major improvement over the prediction based on the visual, stepwise selection of parameters (curve A).

The volumetric response has not been mentioned so far. The strain path is used as input to control the prediction; the success of the volumetric response can be judged by comparing the predicted stress paths with the (applied) constant mean stress paths (Fig. 7(*a*), (*b*)). In detail these paths are of course very sensitive to erratic changes in direction of the (experimentally derived) strain paths, particularly with the visual stepwise selected parameters (curves A). The optimisation procedure is very successful in matching the actual stress changes (curves B).

Discussion and conclusions

The stress path method (Lambe, 1967) seeks to encourage engineers to match laboratory and field stress paths in order to be able to estimate field deformations in a more rational way. Estimation of field deformations is more readily achieved by numerical analysis than by hand calculation, and consideration of stress paths encourages engineers to be aware of the nature of the extrapolation that is implied in the numerical predictions (Wood, 1984). It is a logical extension then to encourage engineers to make their numerical models match the experimental data over the ranges of stress or strain changes that are actually expected to be important. Clearly this will often be an iterative process, with stress paths that emerge from numerical analyses performed for working loads being used to define the range of stress in laboratory tests over which the optimum set of soil parameters should be assessed.

The possibilities of optimisation in parameter selection have been presented here for just one test, to illustrate some of the problems that may emerge. It would normally be preferable to combine data from several tests, either repeating the response on a single path, to provide some information about reliability of experimental data, or in order to increase the volume of relevant stress hyperspace over which data have been gathered. Different tests can be assigned different weights in the optimisation procedure in order to reflect assessments of test quality or relevance to the particular prototype problem.

It will be noted that no suggested optimum values of Γ or N have been quoted. The Cam clay algorithm used here is typical of those used with finite element programs (see, for example, Britto and Gunn (1987)) in that it expects the initial size p'_{0i} of the yield surface to be specified at the same time as the initial effective stresses. The initial specific volume v_i is required in order that strain increments may be calculated from (1)–(4) but the link between v_i, p'_{0i} and initial mean effective stress p'_i through N, λ and κ is not forced:

$$v_i = N - \lambda \ln p'_{0i} + \kappa \ln (p'_{0i}/p') \tag{11}$$

the value of p'_{0i} becomes an optimisation variable, whereas comparison and combination of tests with different consolidation histories requires that N or Γ be used instead. This merely requires a minor program modification.

It would of course be quite unwise to use such an optimisation procedure without interaction with an informed user. The process cannot be allowed to become a 'black box'. The user needs to ensure that the parameters that are chosen are indeed reasonable, and needs to be intelligent in choosing data which cover the appropriate stress level and stress and strain ranges. The objective function for a model like Cam clay has many minima, and it is clearly sensible to seed the optimisation process with initial values which have been deduced from visual interpretation of the experimental observations in the traditional manner.

Nevertheless, releasing the Cam clay parameters from their physical origins, and concentrating the prediction on stress changes of prototype interest, improves the performance of the model. It remains a simple model, requiring a small number of soil parameters, and it may be preferable to tune it to give a good local prediction of response, rather than to tune it to the global response of the soil and still to expect it to perform well locally.

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